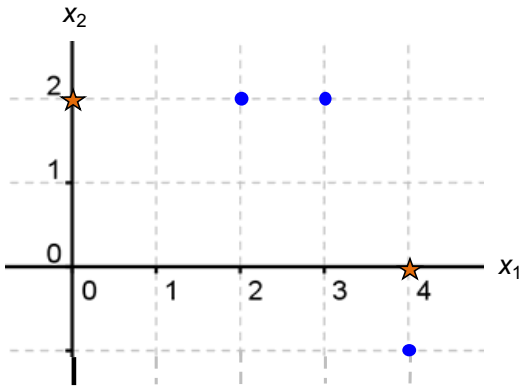
K-means clustering *(find and work with a partner)*

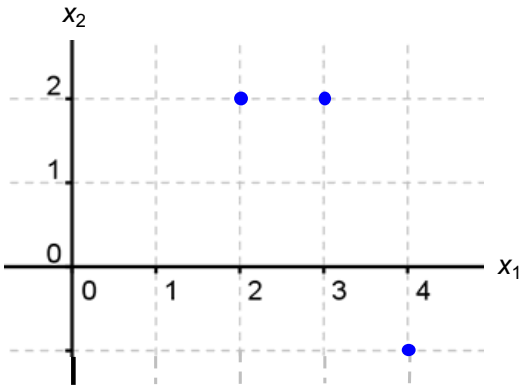
1. Consider the data below with $n = 3$ and $p = 2$. The graph below shows these 3 points (circles), as well as the initial means (stars) for $K = 2$. Here $\vec{\mu}_1^{(1)} = [0, 2]$ and $\vec{\mu}_2^{(1)} = [4, 0]$.

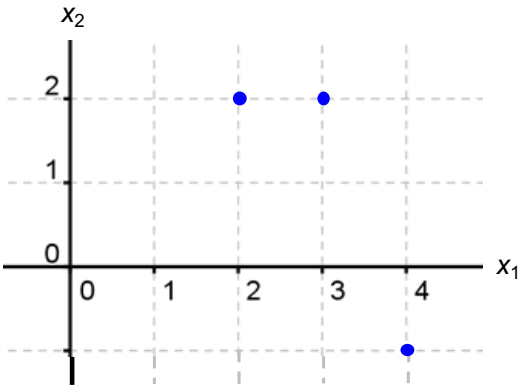
$$\mathbf{X} = \begin{bmatrix} 3 & 2 \\ 2 & 2 \\ 4 & -1 \end{bmatrix}$$



(a) On the graph above, show the cluster membership of each point, based on these initial means. What are $\mathcal{C}_1^{(1)}$ and $\mathcal{C}_2^{(1)}$?

(b) Based on these cluster memberships, what are $\vec{\mu}_1^{(2)}$ and $\vec{\mu}_2^{(2)}$? Draw these two points as stars on the left plot below. This concludes the first iteration of the K-means algorithm.





(c) Based on the new means, draw the new cluster memberships and list $\mathcal{C}_1^{(2)}$ and $\mathcal{C}_2^{(2)}$. Finally, on the right plot above, draw the final means $\vec{\mu}_1^{(3)}$ and $\vec{\mu}_2^{(3)}$ and write out their values.

2. Does the "within cluster sum of squares" (WCSS) always decrease as K (number of clusters) increases?

3. Compute the WCSS for the points above, using $K = 1$, $K = 2$, and $K = 3$.

4. Finally, plot $K = 1, 2, 3$ on the x-axis and WCSS on the y-axis to create an "elbow" plot. What K would you choose in this case?

5. In terms of n, p, K, and T (max number of iterations), what is the runtime of K-means?