## *K*-means clustering

(find and work with a partner)

1. Consider the data below with n = 3 and p = 2. The graph below shows these 3 points (circles), as well as the initial means (stars) for K = 2. Here  $\vec{\mu}_1^{(1)} = [0, 2]$  and  $\vec{\mu}_2^{(1)} = [4, 0]$ .



- (a) On the graph above, show the cluster membership of each point, based on these initial means. What are  $C_1^{(1)}$  and  $C_2^{(1)}$ ?
- (b) Based on these cluster memberships, what are  $\vec{\mu}_1^{(2)}$  and  $\vec{\mu}_2^{(2)}$ ? Draw these two points as stars on the left plot below. This concludes the first iteration of the K-means algorithm.



(c) Based on the new means, draw the new cluster memberships and list  $C_1^{(2)}$  and  $C_2^{(2)}$ . Finally, on the right plot above, draw the final means  $\vec{\mu}_1^{(3)}$  and  $\vec{\mu}_2^{(3)}$  and write out their values.

- 2. Does the "within cluster sum of squares" (WCSS) always decrease as K (number of clusters) increases?
- 3. Compute the WCSS for the points above, using K = 1, K = 2, and K = 3.

4. Finally, plot K = 1, 2, 3 on the x-axis and WCSS on the y-axis to create an "elbow" plot. What K would you choose in this case?

5. In terms of n, p, K, and T (max number of iterations), what is the runtime of K-means?