Linear Regression *(find and work with a partner)*

In the case of *simple* linear regression, for each example we have a single feature x and associated response (output) variable y. Our model is:

$$h_{\boldsymbol{w}}(x) = w_0 + w_1 x$$

1. Name one goal of linear regression. Why would we want to fit a linear model?

2. In simple linear regression, how many features does each example have? How many parameters does our linear model have in this case?

   - # features (p) =

   - # model params =

3. What cost/loss function are we trying to minimize in simple linear regression?

4. *Small example.* Let $n = 2$ and $p = 1$, with the following data:

$$\boldsymbol{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   Plot these two points – what should $\hat{w}_0$ and $\hat{w}_1$ be?

5. This week we derived the solution for simple linear regression:

$$\hat{w}_1 = \frac{\mathrm{Cov}(\boldsymbol{x}, \boldsymbol{y})}{\mathrm{Var}(\boldsymbol{x})} = \frac{\frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^2} \qquad \hat{w}_0 = \bar{y} - \hat{w}_1 \bar{x}$$

   where $\bar{x} = \frac{1}{n}\sum_{i=1}^n x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^n y_i$. Use these equations to compute $\hat{w}_0$ and $\hat{w}_1$ and verify your answer to the previous question.