

Central Limit Theorem example

The central limit theorem (CLT) states that if X_1, X_2, \dots, X_n are drawn from a population with mean μ , finite variance σ^2 , and sample mean \bar{X}_n , then

$$Z = \lim_{n \rightarrow \infty} \sqrt{n} \left(\frac{\bar{X}_n - \mu}{\sigma} \right)$$

is a standard normal distribution. The central limit theorem can be used to estimate test statistics, which can be used to compute p-values. The two key assumptions are that the variance is finite and the sample size is large.

In the following example we flip a coin 80 times, and observe 54 heads. The question is: is this coin unfair?

1. Coin tosses can be thought of as samples from the Bernoulli distribution, where Tails=0 and Heads=1. We can write this as $X \sim \text{Ber}(p)$, where p is the probability that $X = 1$. For a fair coin, what is the expected value of X , i.e. $E[X]$?
2. For a fair coin, what is the variance of X , i.e. $\text{Var}(X)$? Use the fact that $\text{Var}(X) = E[(X - \mu)^2]$, where $\mu = E[X]$.
3. For the example above (54 heads out of 80 tosses), what is the sample mean \bar{X}_n ?
4. Putting this all together and using the CLT, what is the test statistic? (use a calculator)
5. Sketch out the position of the test statistic on a standard normal distribution. Shade the area(s) representing the corresponding p-value.
6. Based on your diagram, do you think this coin is fair?

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